Novel Effects of Light in Non-Hermitian Photonic Systems

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Non-Hermitian photonic systems

- Definition
- Examples
  1. Parity-Time (PT) symmetric systems
     • Phase transition
     • Conservation law
     • Anti-PT?
     • Applications (single-mode laser, lossy amplifier, etc)
  2. Microdisk lasers
     • Power enhancement
     • Mode switching
     • Detecting rotation-induced relativistic effect
Quantum Mechanics

• Basic ansatz: All physical observables correspond to Hermitian operators, including position $x$, momentum $p$, and the Hamiltonian $H$ ("energy" $E$)

$$H = H^\dagger, H \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = E \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}$$

\[\dagger:\text{ complex conjugate (*) plus transpose in the matrix form}\]

$$H = \begin{bmatrix} E_1 & g \\ f & E_2 \end{bmatrix}, \quad H^\dagger = \begin{bmatrix} E_1^* & f^* \\ g^* & E_2^* \end{bmatrix}$$
Quantum Mechanics

• Why we need a Hermitian operator?

Reason: Their expected values are assumed to give the observed physical values, which need to be real numbers.

\[ H \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} = E \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}, \quad E = E^* \]

Implication: The probability density is conserved.

\[ \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}(t = t_1) = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}(t = t_0) \exp(-iEt/\hbar) \]

\[ \Rightarrow \left| \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}(t = t_1) \right|^2 = \left| \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}(t = t_0) \right|^2 \text{ is conserved} \]
Quantum Mechanics

• Question: Can QM deal with decay and amplification?

Short answer: Yes but difficult. It requires that the energy/particle drain and source ("bath") are included in the whole system.

Effective approach: Allowing the operators (especially the Hamiltonian) to be non-hermitian (e.g., $E = E_0 + iE_1$)

\[
\begin{bmatrix}
\Psi_1 \\
\Psi_2
\end{bmatrix}(t = t_1) = \begin{bmatrix}
\Psi_1 \\
\Psi_2
\end{bmatrix}(t = t_0) \exp(-iEt/\hbar)
\]

\[
\left\| \begin{bmatrix}
\Psi_1 \\
\Psi_2
\end{bmatrix}(t = t_1) \right\|^2 = \left\| \begin{bmatrix}
\Psi_1 \\
\Psi_2
\end{bmatrix}(t = t_0) \right\|^2 \exp(-2E_1t) \text{ is not conserved}
\]
Quantum Mechanics

• New Question: Can a non-hermitian operator give real-valued observables?

\[ H \Psi(x, t) = \left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right] \Psi(x, t) \]

\[ = i\hbar \frac{\partial}{\partial t} \Psi(x, t) = E\Psi(x, t) \]

• General condition: Find a transformation such that

\[ O[H\Psi(x, t)] = H\Psi(x, t), \]
\[ O[E\Psi(x, t)] = E^*\Psi(x, t) \]
Quantum Mechanics

\[ i\hbar \frac{\partial}{\partial t} \Psi(x, t) = E\Psi(x, t) \]

Easy to achieve \( O[E\Psi(x, t)] = E^*\Psi(x, t) \): Time reversal \( T (t \rightarrow -t) \), which is the same as taking complex conjugate

\[ H \Psi(x, t) = \left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t) \right]\Psi(x, t) \]

To achieve \( O[H\Psi(x, t)] = H\Psi(x, t) \): \( V(x, t) \) needs to be invariant upon \( O = ZT \)

- **Hermitian case**: \( V(x, t) \) is real; \( Z = 1 \) (i.e., \( O = T \)) is all we need.
- **Non-hermitian case**: \( V(x, t) \) is complex; \( Z \neq 1 \)
Parity-Time (PT) symmetry

\[
Z = P \left( x \to -x \right) ; V(-x, t) = V^*(x, t)
\]

Example of PT-symmetric potential: \( V(x, t) = (ix)^N \)

Bender and Boettcher, PRL 80, 5243 (1998)

- \( N > 2 \): All energies are real
- \( 1 < N < 2 \): finite number of real energy levels
- \( N = 1^+ \): Only the ground state has real energy

Q: How to realize such a potential?
PT-symmetric photonic systems

Paraxial Optics

Paraxial Wave equation

\[ i \frac{\partial E}{\partial z} + \frac{1}{2k} \frac{\partial^2 E}{\partial x^2} + k_0 n(x) E = 0 \]

Propagation constants

Quantum Mechanics

Schrödinger equation

\[ i \hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \psi \]

Energy eigenvalues

Index of refraction $n(x)$

PT-symmetric potential: $n(-x) = n^*(x)$

\[ \Psi(x, t) \propto \exp(-i\omega t + inkz) \]
\[ |\Psi(x, t)| \propto \exp(-\text{Im}[n]kz) \]

- $\text{Im}[n] > 0 \rightarrow$ decay
- $\text{Im}[n] < 0 \rightarrow$ amplification

<table>
<thead>
<tr>
<th>Loss (Im[n]&gt;0)</th>
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<tr>
<td>Gain (Im[n]&lt;0)</td>
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Phase transition in PT-symmetric systems

\[ H_{\mathcal{PT}} = \begin{bmatrix} E_0 + i\kappa & \gamma \\ \gamma & E_0 - i\kappa \end{bmatrix} \]

Bender, Brody, and Jones, PRL 89, 270401 (2002)
Phase transition in PT-symmetric photonic systems

\[ V(x) = \sec h^2(x) + iV_0 \sec h(x) \tanh(x) \]

Phase transition in PT-symmetric photonic systems

Experimental Observation of:
Diffraction dynamics in a PT-lattice
PT-Bloch oscillations
Unidirectional invisibility
PT scattering phase transition

\[ \nabla^2 + \frac{n^2(x)\omega^2}{c^2} \Psi(x) = 0 \]

- General Helmholtz equation;
- Only rely on PT-symmetry;
- Do not reply on the analogue to Schrodinger Eq.
PT scattering phase transition

Chong, LG and Stone, PRL 106, 093902 (2011)

$s$: scattering eigenvalues
Either amplifying or attenuating
⇒ Interferometric amplifier-absorber
PT scattering phase transition

Chong, LG and Stone, PRL 106, 093902 (2011)

$s$: scattering eigenvalues
Either amplifying or attenuating ⇒ Interferometric amplifier-absorber
PT scattering phase transition

Chong, LG and Stone, PRL 106, 093902 (2011)
In higher dimensions PT-symmetric systems with intrinsic degeneracy, an entirely real spectrum of eigenvalues is impossible, unless there are additional discrete symmetry.

LG and Stone, PRX (2014)
PT scattering phase transition

- Generalized point group e.g. Dihedral group $D_{2v} = \{1, v - 1 \text{ rotation}, v \text{ reflections}\}$

With PT-symmetry (and RT-symmetry)

$DT_8 \equiv \{1, PT, P_{\pm \frac{\pi}{4}}, P_{\frac{\pi}{2}} T, R_{\frac{\pi}{2}} T, R_{\pi}, R_{\frac{3\pi}{2}} T\}$

LG and Stone, PRX (2014)
Lasers

- Small
- Low threshold
- On-chip integration

Total internal reflection

Top view

Pump
Singularities in Broken PT phase

Singular points in PT broken phase:

\[ |s_+| \rightarrow \infty \Rightarrow |s_-| \rightarrow 0, \text{ CPA-Laser} \]
PT scattering phase transition

LG and Stone, PRX (2014)

Liang et al. Science (2014)
Anti-PT symmetry? 

\[ n(-x) = -n^*(x) \]
\[ \mu(-x) = -\mu(x) \]
\[ n(-x) = n^*(x) \]

PT symmetry

Real part of index: balanced PIMs and NIMs

Ge and Tureci, PRA 88, 053810 (2013)

Im[n]

Yen et al Science 303, 1494 (04)

Zhang et al PRL 94, 37402 (05)

Dolling et al. Opt. Lett. 31, 1800 (06)
Resonances in an optical cavity

\[ \Delta \omega \]

Frequency

Transmission

\[ q_{m-1}, q_m, q_{m+1} \]

Frequency
Resonances in an anti-PT cavity

Ge and Tureci, PRA 88, 053810 (2013)
Lasing in an anti-PT cavity

Ge and Tureci, PRA 88, 053810 (2013)
Supersymmetry (SUSY) in Non-Hermitian Systems

Original system (A1) SUSY partner (A2)
Micro Lasers

- Small footprint
- Low threshold
- On-chip integration

Peter et al

Total internal reflection
Top view
Power Enhancement using incoherent control

Enhancement of laser power-efficiency by control of spatial hole burning interactions

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