1. EQUIPOTENTIAL SURFACES AND ELECTRIC FIELD LINES

**Objective**

The objective of this laboratory work is to directly measure electric potential produced by electric charges, to plot equipotential lines and electric field lines around electric charges of various configurations and to perform estimation of electric field strength.

**Physical Principles**

Electric charge is fundamental perturbation of free space. Any charge $Q$ distorts space around itself. This distortion, known as **electric potential** $V$, is proportional to the magnitude of the charge $Q$ and, if the charge can be considered as a point (small size) charge, is inversely proportional to the distance $r$ from the charge (Eq. 1):

$$ V = k \frac{Q}{r} $$

where $k$ is the Coulomb constant ($k = 9 \times 10^9$ Vm/C). Locus of points of the same potential is an **equipotential surface**.

The rate of change of electric potential $\Delta V$ over distance $\Delta d$ is known as **electric field** $E$: (Eq. 2):

$$ E = \frac{\Delta V}{\Delta d} $$

Electric field can be revealed by placing another charge $q$ (test charge) in the proximity of the charge $Q$ and measuring force $F$ acting upon it. Then the strength of electric field and its direction is found as the magnitude and direction of the force $F$ exerted on unit test charge (Eq. 3).

$$ E = \frac{F}{q} $$

Electric charges can be of two signs: positive and negative. Like charges repel each other, whereas the charges of opposite signs attract each other. Thus, the electric field created by a positive charge is directed from the charge (direction of the force acting...
upon positive test charge), whereas the electric field created by a negative charge is directed towards the charge (direction of the force acting upon positive test charge). The family of curves, whose tangents point in the direction of the electric field, are known as *electric field line*. Electric field lines are always normal with respect to equipotential surfaces.

The difference of electric potential $\Delta V_{ab}$ between two points $a$ and $b$ equals to work required to move a unit positive charge from point $a$ to point $b$. The absolute electric potential $V_a$ at a point $a$ is defined as the work required to move a unit positive charge from infinity to the point $a$.

For isolated point charges, the equipotential surfaces are spheres, whereas the electric field lines are straight lines (Fig. 1).

For assembly of point charges and non-point charges, equipotential surfaces and electric field lines have more complex shapes see Figs. 2 and 3.
Fig. 2. Two dimensional presentation of electric field lines and equipotential lines of positive and negative charges placed at a short distance one from another. Two points \( a \) and \( b \) between which strength of electric field is measured are shown. The distance \( d_{ab} \) is smaller than the distance between the charges and total length of the electric field line.

Fig. 3. Two dimensional presentation of electric field lines and equipotential lines of two oppositely charged parallel plates. The uniform electric field between the plates is shown by straight parallel electric field lines.

The electric field between two oppositely charged parallel plates placed at a distance much smaller than size of the plates can be considered as uniform (Fig. 3). Note that the electric field in the areas close to the edges of the plates is not anymore uniform.
For uniform electric field, there is a simple relation between the strength of electric field $E$ and the potential difference $V_{ab}$ between points $a$ and $b$ lying on one and the same electric field line (Eq. 4): 

$$E = \frac{V_a - V_b}{d_{ab}}$$

where $d_{ab}$ is the distance between points $a$ and $b$. Although this formula is not strictly correct for non-uniform electric field, it can be used for estimation of strength of electric fields of any configuration (Fig. 2). In this case, however, the distance $d_{ab}$ must be much less than the distance between the charges creating the electric field and/or total length of the electric field line.

Electric field can be created only in non-conductive media, e.g. in vacuum, air, or insulating materials like glass or water. Electric field does not penetrate inside conductors. Thus, inside conductive materials electric field is zero. It is also true for closed hollow conductive objects, e.g. closed metal box, or closed metal cage. Since electric field is zero, electric potential inside conductors and conductive hollow objects is constant (Fig 4).
**Apparatus**

- Conductive paper
- Adhesive copper dots and strips
- Cork board
- Metal push pins
- White paper (8½"x14")
- Carbon paper
- Digital multimeter with probes
- Connecting wires with alligator clips
- 4 x D-size battery holder

![Diagram](image)

**Fig. 5.** Scheme of the experimental set-up showing the electrical connections between conductive paper, battery, multimeter and the probes.

**Experimental Procedure and Calculations**

1. Set up the experiment for two point charges configuration, as in Fig. 4 and 5a.
2. Mark points of equal potential for at least 5 different voltages. For each potential value, include enough points to reasonably determine the shape of the equipotential line.
3. Repeat parts 1 and 2 for the remaining configurations (Fig. 5b and 5c).
4. From the points you have marked, carefully construct the equipotential lines for each charge distribution.
5. Construct the electric field lines. Remember that electric field lines are everywhere perpendicular to equipotential lines.
6. Calculate electric field strength in 3 points of your choice on each graph.
7. Estimate the amount of electric charge on the point electrodes (the configuration on Fig. 6a) using the accumulated data.

Fig. 6. Configurations of charged metal electrodes on conductive paper: (a) two point charges, (b) two parallel plates, (c) closed conductive surface.

**Questions**

1. Is it possible for two different equipotential lines to cross each other? Explain why or why not?
2. Is it possible for two different electric field lines to cross each other? Explain why or why not?
3. Where do the electric field lines begin and end? If they are equally spaced at their beginning, are they equally spaced at the end? Along the way? Why?
4. If you wanted to push a charge along one of the electric field lines from one conductor to the other, how does the choice of electric field line affect the amount of work required? Explain.
5. The potential is everywhere the same on an equipotential line. Is the electric field everywhere the same on an electric field line? Explain.
6. How much work has to be done in order to move an electric charge along an equipotential line?